

Efficient Scheme of Experimental Quantifying non-Markovianity in High-Dimension Systems

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The non-Markovianity is a prominent concept of the dynamics of the open quantum systems, which is of fundamental importance in quantum mechanics and quantum information. Despite of lots of efforts, the experimentally measuring of non-Markovianity of an open system is still limited to very small systems. Presently, it is still impossible to experimentally quantify the non-Markovianity of high dimension systems with the widely used Breuer-Laine-Piilo (BLP) trace distance measure. In this paper, we propose a method, combining experimental measurements and numerical calculations, that allow quantifying the non-Markovianity of a N dimension system only scaled as N^2 , successfully avoid the exponential scaling with the dimension of the open system in the current method. After the benchmark with a two-dimension open system, we demonstrate the method in quantifying the non-Markovianity of a high dimension open quantum random walk system.

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I. INTRODUCTION

No real physical systems could be regarded as a purely closed system, as they are inevitably interacting with environments. Therefore, the dynamic of the open quantum systems is in the central of the fundamental quantum mechanics and quantum information science. [1] Non-Markovianity [2] is the prominent concept in open systems and attracts a lot of attention from theoretical and experimental aspects. [3–10] It has been shown that the non-Markovianity can be exploited as useful resource in quantum technology. For examples: it may be used to improve efficiency of quantum information processing and communication; [4] it is benefit in quantum metrology; [3] and it may be used to improve the security in continuous-variable quantum key distribution, [11] etc. It is therefore important to quantify the non-Markovianity of a quantum system. However, despite of the extensive investigation, characterization and measurement of the non-Markovianity of an open system is still limited to very small systems, [12–14] mostly one-qubit system in experiments. [15]

The measurement base on trace distance, proposed by Breuer, Laine and Piilo (BLP), [6] is one of the most popular definitions for non-Markovianity, and is widely used in theoretical and experiment investigations. [15, 16] To quantify the non-Markovianity by BLP measurement, one needs to find a pair of initial states to maximal a function based on the trace distance. Only when the interactions between the system and the environment are exactly known (the dimension of the whole system

is not too huge), the optimal state pair and the measure of the non-Markovianity can be found numerically. For very limited quantum open systems which can be exactly solved, such as the Jaynes-Cummings model [17] and quantum Brownian motion model, [18] the measure of the non-Markovianity can be analytically found. However, for general quantum open systems where we have no exact information about the interaction between the system and the environment (or the dimension of the whole system is large), the optimal initial state pair can only be found experimentally by scanning the dynamics of the whole initial state spaces which is a tough task even for two-dimension systems. In a typical experiment [19] to quantify the non-Markovianity in a two-dimension quantum open system, in order to achieve a reasonable accuracy, total 5000 states' dynamics were measured. Even worse, the number of scanning states grows exponentially with the dimension of the system. For a system containing two spin-1/2 qubits (which can be viewed as a 4-dimension system), one need to scan about D^4 different initial states to obtain the non-Markovianity, where D is the number of samples for each degree of freedom. Typically, D should be 100 or even more to ensure the accuracy. Therefore even quantification the non-Markovianity of this simple two qubits system is beyond our current experimental ability.

In this work, inspired by the idea of standard quantum process tomography, [20] we propose an efficient method, combining handful experiments (polynomial scaled with the dimension of the system) and the numerical optimization method to measure the non-Markovianity for high dimension systems (The systems containing more than one qubits can be regarded as high dimension systems). This method require no prior information about the interactions between the system and its environment. Due to the linearity of the dynamics of the quantum system,

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we need only experimentally measure the dynamics of some linearly independent states of the system, and the dynamics of the whole state space can be rebuilt by linear combination of these experimental results. We then find the optimal state pair in quantifying non-Markovianity through numerical calculation based on these experiment data. The number of the measurement is scaled as N^2 , where N is the dimension of the quantum system. Using this method, the former intractable non-Markovianity measurement of high dimension system can be easily investigated. After the benchmark on a two-dimension open system, we demonstrate our algorithm to quantify the non-Markovianity of a high dimension open quantum random walk system.

II. METHODS

The BLP measurement of the non-Markovianity [6] is defined on the trace distance of two state ρ_1, ρ_2 , that is, $D(\rho_1, \rho_2) = \frac{1}{2} \text{tr}|\rho_1 - \rho_2|$ where $|A| = \sqrt{A^\dagger A}$. If A is Hermitian, $\text{tr}|A| = \sum_i |\lambda_i|$ is the sum of the absolute value of all the eigenvalue of matrix A . This quantity describe the distinguishability between the states ρ_1 and ρ_2 : if it is zero, the two states are indistinguishable, otherwise, they are distinguishable. Based on this definition, the measure of the non-Markovianity of an open system can be defined as,

$$\mathcal{N} = \max_{\rho_{1,2}(0)} \int_{\delta > 0} dt \delta(t, \rho_{1,2}(0)), \quad (1)$$

where $\delta(t, \rho_{1,2}(0)) = \frac{d}{dt} D[\rho_1(t), \rho_2(t)]$ is the change rate of the trace distance. $\rho_i(t)$, $i=1, 2$ is the density matrix of the open system at time t with the initial state $\rho_i(0)$. The time-integration is extended over all time intervals in which δ is positive, and the maximum should be optimized over all pairs of initial states. Roughly speaking, the integral intervals stand for the time intervals when the information flows back to the system from the environment.

The most difficult task to measure the non-Markovianity is to find a pair of states, ρ_1, ρ_2 that maximize Eq. (1). Generally, it need experimentally scan the state pairs in the whole parameter space. Some simplifications can be made. [19, 21] It has been rigorously proven that the optimal states pair should be on the boundary of the physical state space and the states pair are orthogonal each other. Therefor, we can only scan the boundary of physical state space (For two-dimension case, these states are pure states). Another simplification to the measure was demonstrated in Ref. 19, which illustrated that the measure can be obtained efficiently in an arbitrary neighborhood of any fixed state in the interior of the state space. That is, it needs only scan one state of the pair in the physical state space. This can dramatically reduce the experimental work. However, the number of the experiments is still too large

and will exponentially increase with the dimension of the systems. Therefore, the non-Markovianity in higher dimension is still intractable to quantify with the current method. Now we introduce another scheme to simplify the experimental quantification of non-Markovianity in an open system which make the high dimension system reachable.

Following the idea of the quantum process tomography, [20] the state of a quantum open system with N dimension can be expressed as $N \times N$ density matrix. Any density matrix can be expanded by N^2 linear independent bases,

$$\begin{aligned} \rho_{mn}^x &= (|m\rangle\langle n| + |n\rangle\langle m|)/2, \quad (m > n) \\ \rho_{mn}^y &= i(|m\rangle\langle n| - |n\rangle\langle m|)/2, \quad (m > n) \\ \rho_m^0 &= |m\rangle\langle m|, \end{aligned} \quad (2)$$

where $|m\rangle$ ($m = 1, 2, \dots, N$) is the basis vector of the system. The operators ρ_{mn}^x (ρ_{mn}^y) play the similar role of the pauli matrices σ^x (σ^y) in two-dimension systems. Without loss of generality, we assume that the system and the environment is in a product state at the initial time $t = 0$, i.e. $\rho(0) = \rho_s(0) \otimes \rho_e(0)$, where $\rho(t)$, $\rho_s(t)$, $\rho_e(t)$ are the density matrices of the whole system (system+enviroment), the quantum system and the environment at time t , respectively. Using the above introduced bases, the state of the open system at any time t can be written as:

$$\rho_s(t) = \sum_{m>n} a_{mn}^x \rho_{mn}^x(t) + \sum_{m>n} a_{mn}^y \rho_{mn}^y(t) + \sum_m a_m^0 \rho_m^0(t), \quad (3)$$

where a_{mn}^x , a_{mn}^y , a_m^0 are time independent constants determined by the initial states. $\rho_{mn}^i(t)$ $i = x, y, 0$ is the dynamics of the bases. It suggests that the dynamics of the open system can be completely determined by the dynamics of the bases.

In experiments, the dynamics of the N^2 bases can be obtained by the following procedure:

1. Prepare the initial states of the system to $|m\rangle$, where $m=1, 2, \dots, N$. $|m\rangle$ can be any set of complete and orthogonal vector bases of the system. By measuring the dynamics of the open system, we obtain $\rho_m^0(t)$.
2. Prepare the initial states of the system to $(|m\rangle + |n\rangle)/\sqrt{2}$ ($m > n$), measuring the dynamics of the open system. We obtain $\rho_{mn}^x(t) + \frac{1}{2}(\rho_m^0(t) + \rho_n^0(t))$.
3. Prepare the initial states of the system to $(|m\rangle + i|n\rangle)/\sqrt{2}$ ($m > n$), measuring the dynamics of the open system. We obtain $-\rho_{mn}^y(t) + \frac{1}{2}(\rho_m^0(t) + \rho_n^0(t))$.

We therefore have the dynamics of the N^2 bases of the open system. Using the dynamics of these bases, the BLP measure of the non-Markovianity of the open system can be achieved by numerically optimizing the parameters, a_{mn}^x , a_{mn}^y and a_m^0 , of the initial states by computer using

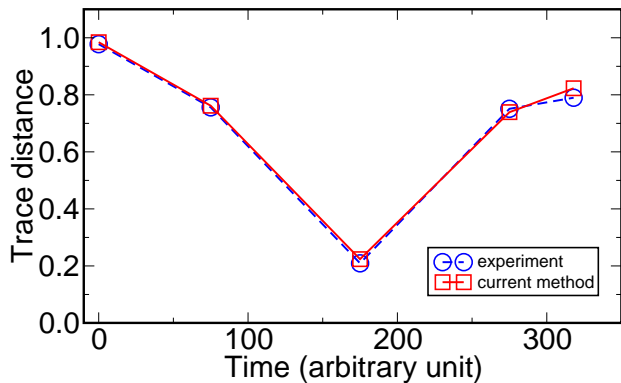


FIG. 1: (Color online) Comparing the trace distance directly measured from experiment and the results obtained from our method. The blue circles show the trace distance as function of time of the experimentally determined optimal state pair, by scanning the whole state space. The red squares show the trace distance of the optimal pair as function of time, obtained by four basis dynamics followed by numerical optimization, as described in the main text.

Eq. (1), and Eq. (3) through deepest descent algorithm. The nice scale of this method make it possible to apply for the high-dimension system which is intractable for the traditional method. It is worth noting that the basis introduced here is not unique. Any N^2 such linear independent states are enough for the procedure. The simplifications introduced in Ref. 21 and Ref. 19, can also be used to reduce the numerical optimization efforts in this method.

III. RESULTS AND DISCUSSION

A. Benchmark for the two-dimension system

To demonstrate the power of our scheme, we first make benchmark tests on the well studied two-dimension open system. In the typical experiment, [19] the two-dimension open quantum system is provided by the polarization of a photon which coupled to the environment through its frequency degree. To quantify the non-Markovianity of this system, 5000 different states in the Bloch surface have been scanned to find the optimal pair in Eq.(1). The conclusion in [21] has been used to simplify the experiment, which states that the optimal state pairs are orthogonal pure states, and therefore, function \mathcal{N} in Eq.(1) only depends on the angles between two states on the Bloch surface.

A 2×2 density matrix can be expanded by identity matrix I and pauli matrixes $\sigma_x, \sigma_y, \sigma_z$ as $\rho_s = \frac{1}{2}(I + \vec{a} \cdot \vec{\sigma})$, where vector \vec{a} is on the surface of the Bloch sphere. Using our method, non-Markovianity measure of this open system can be determined by the dynamics of the pauli matrices, i.e. $\sigma_i(t)$, where $i=x, y, z$ which can be obtained from the dynamics of the four initial states, $|1\rangle$,

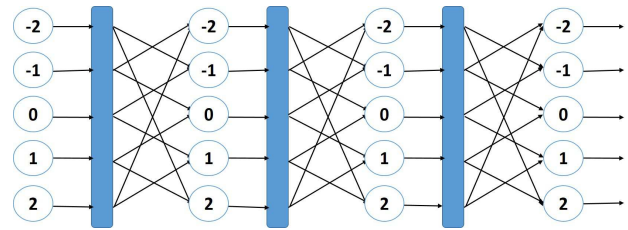


FIG. 2: (color online) Cartoon of the 3-step open quantum walk system, with $X=2$. The blue blocks denote the polarized beam splitter. This quantum walk system includes both the lattice (location of the photon) and the coin (polarization) which is a intrinsically high-dimension system.

$|-1\rangle$, $\frac{1}{\sqrt{2}}(|1\rangle + |-1\rangle)$ and $\frac{1}{\sqrt{2}}(|1\rangle + i|-1\rangle)$ (where $|-1\rangle$ and $|1\rangle$ are two eigenvectors of the polarization photon).

Fortunately, the dynamics of these four initial states can be directly taken from the experimental data in Ref. 19. Using these data, we can completely determine the dynamics of any initial states using Eq. (3). We then numerically find the optimal initial states pair, and the non-Markovianity of this system. The trace distance for the optimal initial states pair as a function of time obtained using our scheme is compared to the directly measured one in Fig. 1. As we see they are in excellent agreement. The non-Markovianity obtained from our method is 0.6, which are very close to the measured value 0.58, both are in good agreement with the theoretical value 0.59. The error in our method is due to the error in the measurement of the 4 basis states.

B. Open quantum walk system

With the confidence of the method in two-dimension open system, we apply this method to a high-dimension open system which can not be reached previously. Here, we demonstrate our method to qualify the non-Markovianity of an one-dimension open quantum walk (QW) system. [22–24] which is intrinsically a high-dimension system due to the ansatz coin. QW system is a generally interested system in quantum information, which has a lot of application in quantum computation. It has been shown that it is a nice tool to find new quantum algorithm and it can be used to constitute a universal model of quantum computation. [25, 26] In addition, QW has been experimentally realized in several different systems. [27–31] The open QW has attracted a lot of attention recently. [22–24]

Here we study the discrete-time QW on a one-dimension lattice. The particle is located at one site at the beginning. At each step, it can move either to the left or to the right which is determined by the state of a coin: $|L\rangle$ (move left) or $|R\rangle$ (move right). In quantum walk, the state of the coin can be a superposed state. Therefore, the state of the whole QW system (including the particle

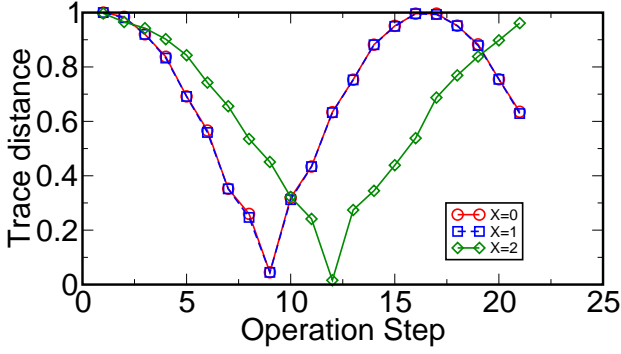


FIG. 3: (Color online) The trace-distances as functions of time for the optimal state pairs obtained using our method for $X=0$ (red line), 1 (blue line) and 2 (green line).

in the lattice and the coin) is $|\psi\rangle = \sum_{x,d} C_{x,d}|x\rangle|d\rangle$ (in open QW, it should be a density matrix) where $x = 0, \pm 1, \pm 2, \dots$ are the location of the particle and $d = L, R$ are the state of the coin. The operator to make up a single step of the QW can be defined as: $W = TC$, where T is the shift operator and defined as $T = \sum_j |j-1\rangle\langle j| \otimes |L\rangle\langle L| + |j+1\rangle\langle j| \otimes |R\rangle\langle R|$, C is the Hadamard coin operator defined as $C = \frac{1}{\sqrt{2}}(|L\rangle\langle L| + |L\rangle\langle R| + |R\rangle\langle L| - |R\rangle\langle R|)$. The state of the system after N_{step} steps with the initial state $|\psi_0\rangle$ can be obtained as $W^{N_{step}}|\psi_0\rangle$.

The discrete-time QW can be implemented with single photon through an array of beam splitters in which the coin states are mimicked by the polarization degrees of freedom. For the open QW, the environment can be introduced through the coupling between the frequency and the polarization degrees of freedom of the photon similar to the method used in Ref. 19. In this case, the single step operator in the open QW can be modified as:

$$U_{step} = U_{\delta t}TC, \quad (4)$$

$$U_{\delta t} = \int d\omega \sum_{p=H,V} e^{in_p\omega\delta t_p} |p\rangle\langle p| \otimes |\omega\rangle\langle\omega|, \quad (5)$$

where T and C are shift operator and Hadamard coin operator defined before. $U_{\delta t}$ couples the polarization(H or V) and environment to give non-Markovianity. n_p is the index of refraction for different polarization state. $\delta t_p = \frac{L}{v_p}$ is the time that operation TC takes place where $L \sim 0.5mm$ is the thickness of the beam splitters and v_p is velocity of light with polarization p in the splitters. $|\omega\rangle$ is the environment state. For convenience, we take the environment as a delta function, that is $|\omega\rangle = \frac{1}{\sqrt{2}}[\delta(\omega - \omega_1) + \delta(\omega - \omega_2)]$. And $\omega_1 = \Omega - \omega_0$, $\omega_2 = \Omega + \omega_0$. We take the value of the parameters from the Ref. 19: $\omega_0 = 7.2 \times 10^{12}s^{-1}$, $\Omega = \frac{2\pi c}{\lambda}$, where $\lambda=780$ nm, thus $\Omega = 2.4166 \times 10^{15}s^{-1}$. $n_H = 1.554$, $n_V = 1.545$, $\delta t_H = Ln_H/c \sim 1.036 \times 10^{-11}s$ and $\delta t_V \sim 1.030 \times 10^{-11}s$. In addition, we use the periodic boundary condition for the system, i.e. $|x_{min}-1\rangle = |x_{max}\rangle$, $|x_{max}+1\rangle = |x_{min}\rangle$ (see

TABLE I: Comparing the number of initial states to measure between the present method and the direct scanning method to obtain the non-Markovianity of the quantum walk system. For the direct method, we assume 100 initial states to measure for each degree of freedom. The non-Markovianity of the system is calculated using the present method.

number of initial states			
X	Direc method	Present method	non-Markovianity
0	100^2	4	0.9512
1	100^6	36	0.9510
2	100^{10}	100	0.9428

Fig.2), and we allow the system evolves for 20 steps.

For convenience, we set the location of the particle $x \in [-X, X]$, then the total dimension of the system is $N=2(2X+1)$. To experimentally measure the non-Markovianity of this system, we need choose N^2 linearly independent initial states. As introduced before, we implement experiments to get the dynamics of the following initial states:

$$|\psi\rangle = |x\rangle|d\rangle \quad (6)$$

$$|\psi\rangle = \frac{1}{\sqrt{2}}(|x_1\rangle|d_1\rangle) + |x_2\rangle|d_2\rangle, \quad (7)$$

$$|\psi\rangle = \frac{1}{\sqrt{2}}(|x_1\rangle|d_1\rangle) + i|x_2\rangle|d_2\rangle \quad (8)$$

here $x, x_1, x_2 = -X, \dots, X$ and $d, d_1, d_2 = H, V$. With the experimental data of these initial states, the dynamics of any initial state can be linearly constructed, and the optimal state pair can be found by numerically searching the configuration space.

Because we do not have the experimental data at hand, we study the dynamics of the QW system numerically using the exact diagonalization method. We first calculate the dynamics of the bases, then search the optimal initial states pair numerically described in previous paragraph, and determine the non-Markovianity. We collect the results of the trace distances of the optimal state pair for $X=0, 1, 2$ in Fig. 3. The non-Markovianity for $X=0, 1, 2$ are given in Table I. Actually, the result can be further confirmed via directly exact diagonalization method in which we obtain exactly the same results.

We also compare the number of the initial states experimentally necessary to determine the non-Markovianity of the open QW system in Table I. The number scales as 100^N , where $N=4X+2$ if previous direct scanning method is used. Clearly it is impossible to experimentally determine the non-Markovianity of the system for $X > 0$ using this method. However, by using our method, in which the number of initial states is only N^2 , we are able to measure the non-Markovianity of much larger systems.

IV. SUMMARY

We have introduced an experimental method to quantify the non-Markovianity of high-dimension open quantum system. In our method, the scaling of the experiment is only N^2 which is dramatically reduced from exponential scaling of the conventional method. Therefore, the system which is intractable by the former method can be easily reached with the current method. After the benchmark with the well studied two-dimension open system, we demonstrate the method to the high

dimension open quantum walk system. This method therefore opens up a new path to experimentally study the non-Markovianity of high dimension open quantum system, which was impossible previously.

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- [1] H. P. Breuer and F. Petruccione, *The Theory of Open Quantum Systems* (Oxford University Press, Oxford, 2007).
 - [2] R. Angel, H. Susana F., and P. Martin B., Rep. Prog. Phys. **77**, 094001 (2014).
 - [3] A. W. Chin, S. F. Huelga, and M. B. Plenio, Phys. Rev. Lett. **109**, 233601 (2012).
 - [4] B. Bylicka, D. Chruściński, and S. Maniscalco, ArXiv e-prints (2013), 1301.2585.
 - [5] R. Vasile, S. Maniscalco, M. G. A. Paris, H. P. Breuer, and J. Piilo, Phys. Rev. A **84**, 052118 (2011).
 - [6] H. P. Breuer, E. M. Laine, and J. Piilo, Phys. Rev. Lett. **103**, 210401 (2009).
 - [7] E. M. Laine, J. Piilo, and H. P. Breuer, Phys. Rev. A **81**, 062115 (2010).
 - [8] R. Vasile, S. Olivares, M. G. A. Paris, and S. Maniscalco, Phys. Rev. A **83**, 042321 (2011).
 - [9] S. Luo, S. Fu, and H. Song, Phys. Rev. A **86**, 044101 (2012).
 - [10] S. Lorenzo, F. Plastina, and M. Paternostro, Phys. Rev. A **88**, 020102 (2013).
 - [11] R. Vasile, S. Maniscalco, M. G. A. Paris, H. P. Breuer, and J. Piilo, Phys. Rev. A **84**, 052118 (2011).
 - [12] X. M. Lu, X. G. Wang, and C. P. Sun, Phys. Rev. A **82**, 042103 (2010).
 - [13] Z. Y. Xu, W. L. Yang, and M. Feng, Phys. Rev. A **81**, 044105 (2010).
 - [14] E. M. Laine, H. P. Breuer, J. Piilo, C. F. Li, and G. C. Guo, Phys. Rev. Lett. **108**, 210402 (2012).
 - [15] B. H. Liu, L. Li, Y. F. Huang, C. F. Li, G. C. Guo, E. M. Laine, H. P. Breuer, and J. Piilo, Nature Phys. **7**, 931934 (2011).
 - [16] B. H. Liu, D. Y. Cao, Y. F. Huang, C. F. Li, G. C. Guo, E. M. Laine, H. P. Breuer, and J. Piilo, Sci. Rep. **3**, 1781 (2013), 1208.1358.
 - [17] B. J. Dalton, S. M. Barnett, and B. M. Garraway, Phys. Rev. A **64**, 053813 (2001).
 - [18] F. Intravaia, S. Maniscalco, and A. Messina, Phys. Rev. A **67**, 042108 (2003).
 - [19] B. H. Liu, S. Wißmann, X. M. Hu, C. Zhang, Y. F. Huang, C. F. Li, G. C. Guo, A. Karlsson, J. Piilo, and H. P. Breuer, Sci. Rep. **4**, 6327 (2014).
 - [20] I. L. Chuang and M. A. Nielsen, J. Mod. Opt. **44**, 2455 (1997).
 - [21] S. Wißmann, A. Karlsson, E. M. Laine, J. Piilo, and H. P. Breuer, Phys. Rev. A **86**, 062108 (2012).
 - [22] S. Attal, F. Petruccione, C. Sabot, and I. J. Sinayskiy, Stat. Phys. **147**, 832 (2012).
 - [23] M. Bauer, D. Bernard, and A. Tilloy, Phys. Rev. A **88**, 062340 (2013).
 - [24] N. Konno and H. J. Yoo, Stat. Phys. **150**, 299 (2013), 1209.1419.
 - [25] A. M. Childs, Phys. Rev. Lett. **102**, 180501 (2009).
 - [26] S. E. Venegas-Andraca, Quant. Inf. Proc **11**, 1015 (2012).
 - [27] A. Schreiber, K. N. Cassemiro, V. Potocek, A. Gabris, P. J. Mosley, E. Andersson, I. Jex, and C. Silberhorn, Phys. Rev. Lett. **104**, 050502 (2010).
 - [28] M. Karski, L. Frster, J. M. Choi, A. Steffen, and et al, Science **325**, 174 (2009).
 - [29] P. Alberto, L. Mirko, C. F. Jonathan, and et al, Science **329**, 1500 (2010).
 - [30] H. Schmitz, R. Matjeschk, C. Schneider, J. Glueckert, M. Enderlein, T. Huber, and T. Schaetz, Phys. Rev. Lett. **103**, 090504 (2009).
 - [31] M. A. Broome, A. Fedrizzi, B. P. Lanyon, I. Kassal, A. Aspuru-Guzik, and A. G. White, Phys. Rev. Lett. **104**, 153602 (2010).